TITLE: Interacting Solitons in a Nonneutral Plasma

Interacting Solitons in a Nonneutral Plasma

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Abstract. We have been continuing our study of solitons and nonlinear interactions in a pure electron plasma. The solitons are grown from large amplitude normal-mode oscillations. When grown in this way, it is convenient to divide the resulting wave into two parts: localized soliton-like structures and global normal-mode-like waves. These two parts interact weakly with each other and cause various interesting effects. When we launch two solitons by using the $n_z = 2$ mode, the normal mode part quickly converts to an $n_z = 1$ mode and causes one of the solitions to disappear. Later, when only one solition exists, the $n_z = 1$ mode is locked to it. Because the soliton speed is faster than the linear wave speed, this increases the frequency of the normal-mode part of the system. It appears that when the soliton amplitude drops below some critical value, the two parts of the system decouple and propagate separately.

INTRODUCTION

We have been studying electrostatic Trivelpiece-Gould modes in a nonneutral plasma confined in a Malmberg-Penning trap[1]. Our plasma is 60 cm long and about 2 cm in radius. The plasma temperature is about 1 eV[2]. We have previously reported work showing that at large amplitude these waves have some of the properties of solitons[3], but that work was limited by the fact that signals from only two of the wall sectors could be recorded on a single shot. We synchronized the signals from different rings by recording a common ring on all of the shots. This limited our ability to evaluate events that were not reproducible from shot to shot. In this study we were able to record all of the wall signals on a single shot basis and therefore study the interesting events that were not reproducible between shots.

GROWTH OF SOLITONS FROM NORMAL MODES

A soliton is a wave in a dispersive medium that is large enough that nonlinear steepening effects just balance the dispersive spreading, causing it to propagate unchanged. Solitons occur in many physical situations[4].

The cold fluid equations for a plasma in a cylinder can be manipulated, making some assumptions, into the form of the first integral of the Korteweg-deVries equation[5]. This means that these solitons should have the properties of the well known solutions of that equation.

CP606, *Non-Neutral Plasma Physics IV,* edited by F. Anderegg et al.

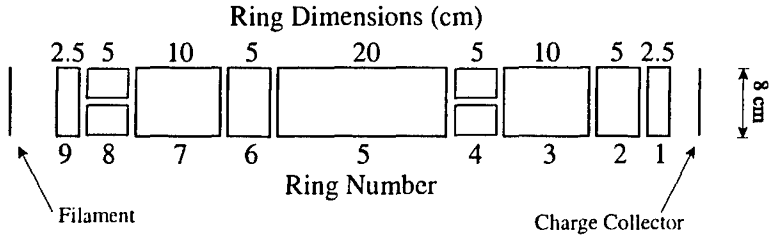

FIGURE 1. Characteristics of the wall rings in our trap.

Solitons in a nonneutral plasma have been created in two ways. One is to put a large potential step on a confining ring[6, 7]. This requires a relatively large voltage (tens to hundreds of volts.) Another way is to create them from normal modes[5]. With this second method, which is the one used in this paper, the confining potentials of the plasma are oscillated at a frequency slightly (about 30 kHz) above the normal mode frequency of the plasma (about 4.22 MHz). This has the effect of repeatedly hitting the pulses at just the right time with a small voltage, building them up to large pulses.

The number of solitons created from a given normal mode is initially equal to n_z (the mode number of the the standing wave, which is equal to the number of half-wavelengths in the plasma.) In numerical simulations the pulses have the characteristic sech^2 shape of KdV solitons[5]. The fact that the driving frequency has to be above the normal mode frequency is an indication that the solitons are traveling faster than the linear wave speed of the plasma. If they traveled at the linear wave speed, they should travel one wavelength in one period and have a frequency equal to the normal mode frequency.

EXPERIMENTAL SETUP

These signals are detected by the image charge induced in the wall sections of our Malmberg-Penning trap. Figure 1 shows the lengths, positions and identification numbers of the rings in our trap. We usually recorded data on rings 3, 5, 6 and 7. With low amplitude oscillation, we just get normal mode oscillations. The characteristics of normal mode oscillations are that the signals are sinusoidal in time with a fixed 0 or 180 degree phase difference between them. The relative amplitudes are also fixed by the identity of the mode and the geometry of the wall sections. With a larger amplitude drive, you see the negative bump of the soliton as it passes underneath each ring. These characteristics are illustrated in Fig. 2. Part (A) shows a normal mode oscillation, while part (B) of the figure shows two counterpropagating solitons. The presence of two solitons can be inferred from the fact that the bumps on rings 3 and 7 are in phase and that ring 5 shows a bump which is roughly twice as big as on the other rings, since both solitons are under it at the same time. Part (C) shows a later time when one of the solitons has disappeared, as will be discussed later. Note that the time scale on part (C) is twice as long as on the other plots. In this plot the signal on ring 5 is closer in size to that on rings 3 and 7. The sequencing of the peaks through the rings can clearly be followed in this plot.

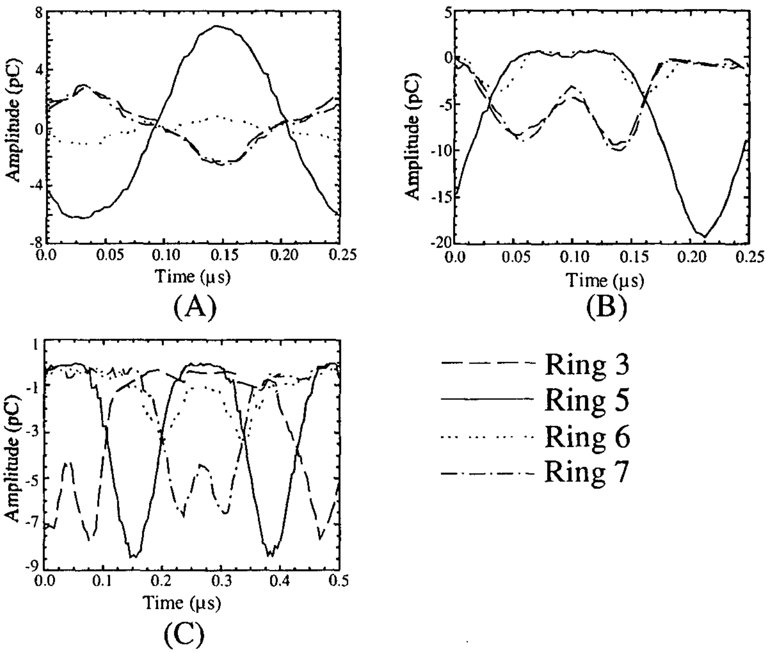

FIGURE 2. Signals for normal modes and solitons. Part (B) shows the signals recorded for a small amplitude oscillations with all of the characteristics of a normal mode oscillation. Part (B) shows the signals when two counterpropagating solitons are present in the system. Part (C) shows the signals when one soliton is present in the system.

EXPERIMENTAL RESULTS

Figure 3 shows the signal recorded on ring 5 as a function of time. The time interval when the system goes from two solitons to one is shown in the first shaded area. The characteristic decrease in amplitude on this ring by a factor of two can clearly be seen. The second shaded region shows the time interval when the second soliton disappears and the signal becomes indistinguishable from a normal mode signal. It appears that both of these events occur because of interactions between the normal modes and the propagating solitons.

In order to describe the signals that we see in all of our rings, it is necessary to break the signal into two parts: a global, sinusoidal in space and time, normal mode part and one or two localized soliton parts. We can calculate and measure the sensitivity of each ring to the two lowest normal modes.

We use a nonlinear least-squares fitting algorithm to fit our four ring signals to these normal mode and soliton parts. Figure 4 shows a typical fit of ring 6 data to two solitons and two normal modes. The normal mode part comes from all four rings together and the soliton parts are derived from just this ring's signal. Note that we describe the signals on these rings as superpositions of these different parts. This is not completely accurate since the solitons are nonlinear waves, but it seems that the interactions are small enough

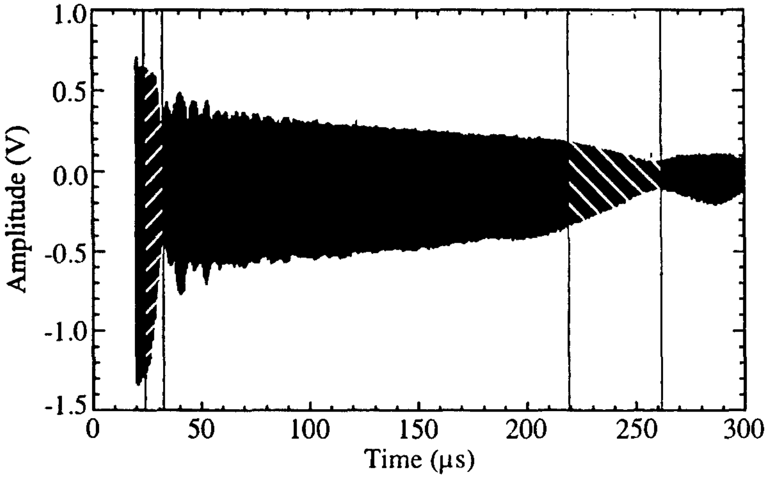

FIGURE 3. Signal recorded on ring 5 as a function of time. The first shaded region shows the time interval when one of the solitons is disappearing. The second shaded region shows the time when the second soliton also goes away.

that this works fairly well, except possibly when the two solitons pass through each other.

The first interaction between the normal modes and the solitons occurs when one of the solitons disappears. The initial state of the plasma is with two equal solitons and a moderate amount of the $n_z = 2$ normal mode. Shortly into the shot this mode undergoes mode conversion from $n_z = 2$ to $n_z = 1$. It appears to be the interaction with this $n_z = 1$ mode that causes one soliton to disappear. As can be seen in figure 4, one of the solitons hits the end of the plasma (at the time of the dip between the two peaks) while the $n_z = 1$ mode is rising, while the other hits while it is falling. One shrinks while the other grows. Figure 5 shows the signal on ring 3 as one soliton slows down and widens as its amplitude decreases. This disappearance takes place over a period of 10 to 20 bounce periods of the soliton.

After one of the solitons disappears, the remaining soliton and the normal mode lock into a fixed phase relationship that persists for hundreds of bounces. Interestingly, the frequency of the normal mode is raised by this interaction with the soliton. The width and frequency of the soliton do not change significantly during this time. At some time later, which depends on the specific shot, the remaining soliton also disappears fairly rapidly, although much slower than the disappearance of the first. It appears to occur on a timescale of 50-60 bounce periods. As shown in figure 6, the characteristics of this disappearance are that normal mode frequency drops to its unperturbed value and the soliton rapidly slows down and widens until its width is essentially a half wavelength of the normal mode. At that point it appears to become part of the normal mode. The soliton and the normal mode also dephase from each other somewhat during this time period, as if they have become decoupled. It is unclear exactly what is the trigger for this series of events. The soliton amplitude is slowly decreasing as it bounces. It appears

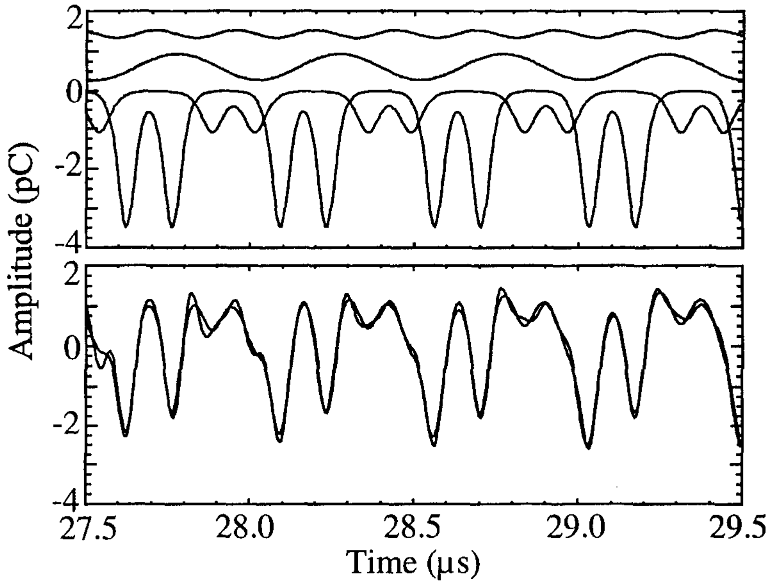

FIGURE 4. Least squares fit of ring 6 data to two solitons and two normal modes. The bottom trace shows both the data and the fit. The top trace shows the different components of the fit. The normal mode signals have been offset vertically for clarity.

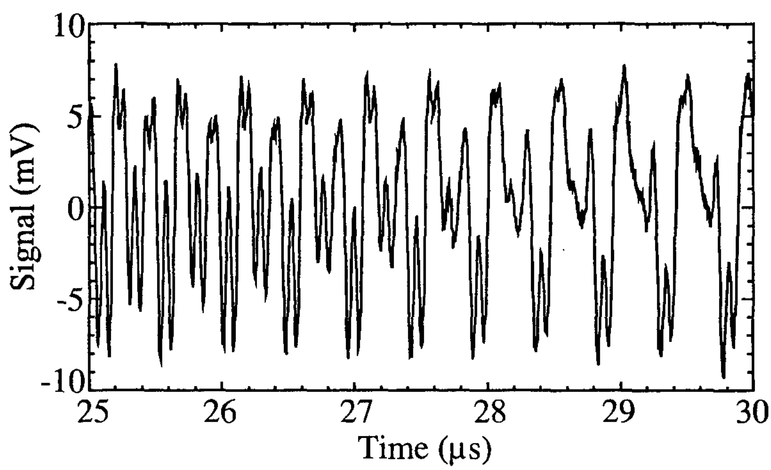

FIGURE 5. When the $n_z = 1$ mode grows, one of the solitons decreases in amplitude and velocity and disappears from the system.

that there is a minimum soliton amplitude necessary to force the coupling between the soliton and the normal mode. There are some indications that this minimum amplitude depends on the frequency shift between the coupled system and the unperturbed normal

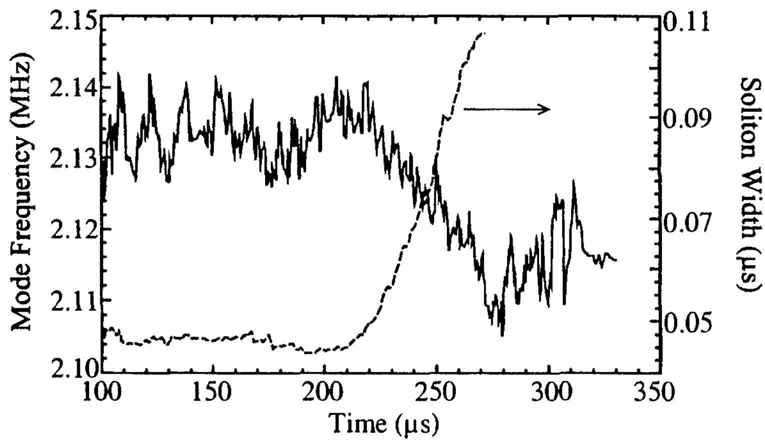

FIGURE 6. When the soliton amplitude gets small enough, the normal mode decouples from the soliton and its frequency drops to its natural frequency. During the same period the width of the soliton increases until it is about one half wavelength of the normal mode.

mode. We do not yet have enough data to verify this.

CONCLUSIONS

When solitons are grown from normal modes in a nonneutral plasma, some of the normal mode character persists underneath the solitons. Interactions between these normal modes and the solitons appear to cause the disappearance of one of the solitons at early times and of the other at much later times, although the mechanisms of the two disappearances appear to be different.

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